# Stochastic faulty actuator-based reliable control for a class of interval time-varying delay systems with Markovian jumping parameters

Zhou Gu<sup>1,2</sup>, Dong Yue<sup>3, \*, †</sup>, Daobo Wang<sup>2</sup> and Jingliang Liu<sup>4</sup>

<sup>1</sup>School of Power Engineering, Nanjing Normal University, Nanjing, Jiangsu, People's Republic of China <sup>2</sup>College of Automation Engineering, Nanjing University of Aeronautics Astronautics, Nanjing, Jiangsu, People's Republic of China

<sup>3</sup>Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, People's Republic of China

<sup>4</sup>College of Information Science and Technology, Donghua University, Shanghai, People's Republic of China

#### SUMMARY

The paper addresses the problem of a reliable control for interval time-varying delay system with Markovian jumping parameters. A new practical actuator fault model, by assuming that the actuator fault obeys a certain probabilistic distribution, is considered. Delay-dependent conditions for the solvability of these problems are obtained via new parameter-dependent Lyapunov function. The closed-loop systems are stochastically stable not only when all actuators are operational, but also in case of some actuator failures. Numerical examples are given to illustrate the effectiveness of the proposed design method. Copyright © 2010 John Wiley & Sons, Ltd.

Received 14 August 2009; Revised 21 January 2010; Accepted 7 April 2010

KEY WORDS: reliable control; stochastic actuator-failure; interval time-varying delay; Markovian jump systems

## 1. INTRODUCTION

A surge of interests in studying the class of Markov jump linear systems (MJLS) has been observed for the past decades. The MJLS are dynamical systems subject to abrupt variations in their structures. Since MJLS are natural to represent dynamical systems that are often inherently vulnerable to component failures, sudden disturbances, change of internal interconnections, and abrupt variations in operating conditions, they are an important class of stochastic dynamical systems [1–3] and the references therein.

Time delays are inherent features of many physical processes, which are often the main cause for instability and poor performance of a control system. Therefore, time-delay systems have been studied in the past years and various research topics on delay systems have been investigated [4–7]. Recently, Markovian systems with time delays have been considered. Sufficient conditions

<sup>\*</sup>Correspondence to: Dong Yue, Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, People's Republic of China.

<sup>&</sup>lt;sup>†</sup>E-mail: medongyue@163.com, medongy@vip.163.com

Contract/grant sponsor: Natural Science Foundation of China (NSFC); contract/grant numbers: 60704024, 60904013, 60774060

for delay-independent mean-square stochastic stability (MSSS) [8–10] and for delay-dependent MSSS [11–13] were obtained.

However, all the aforementioned results are under a full reliability assumption that all actuators are operational. In fact, actuators play a very important role in control systems, which are responsible for transforming the controller output to the plant, how to preserve the closed-loop control system performance under actuator fault condition will be more meaningful. In practical situations, the actuator completely failure or partial failure often occurs in the real world. The main task of this study is to design a controller such that the closed-loop system can maintain stability and good performance, not only when all control components are operational, but also in case of existing some abnormal actuators including fully outages. To the best of our knowledge, there are very few papers dealing with the reliable control for the stochastic time-delay systems with Markovian jumping parameters. This motivates the development of the so-called reliable control theory.

Over the past few decades, the study of reliable control problem becomes more and more practically meaningful and has attracted considerable attention [14–20]. It is noted that the reliable controller design methods in the aforementioned literatures are all based on the assumption that control component failures are modeled as outages, i.e. when a failure occurs, the actuators signal simply becomes zero. However, it cannot represent actuator-failure exactly. The actuator may not be a complete failure, that is, the scale factor  $\xi_i = 0$  is the simplest of special cases. In practical systems, because of actuators aging, zero shift, Electromagnetic Interference, nonlinear amplification in different frequency fields and so on occur. It will be more reasonable and meaningful if the fault scale factor obeys a certain probabilistic distribution in an interval. Up to the authors' knowledge, the stabilization problem for MJLS with probabilistic actuators fault has not been investigated in the open literatures, which motivates us to the further study.

In this paper, we consider the problem of reliable control for a class of continuous-time Markovian jump systems with interval time-varying delay and stochastic failure. The main contributions of this paper are: (1) A more general fault model is adopted for actuator failures, which satisfies a certain probabilistic distribution. (2) To obtain a less conservative results, a new Lyapunov function is constructed, which includes the lower and upper delay bound of interval time-varying delay. Based on this, one splits the item  $\int_{t-\tau(t)}^{t} x^{T}(s)T(r_t)x(s) ds$  into two parts to deal with, respectively, and uses the convexity of the matrix functions to avoid the conservative caused by enlarging  $\tau(t)$ to  $\tau_M$  in the deriving results. Then, by using the linear matrix inequality (LMI) method, sufficient conditions are derived to ensure the existence of the controller which is characterized by the solution to a set of LMIs, Illustrative examples are exploited to demonstrate the applicability of the proposed design approach.

*Notation*:  $\mathbb{R}^n$  denotes the *n*-dimensional Euclidean space,  $\mathbb{R}^{n \times m}$  is the set of real  $n \times m$  matrices, *I* is the identity matrix of appropriate dimensions,  $\|\cdot\|$  stands for the Euclidean vector norm or spectral norm as appropriate. The notation X > 0 (respectively, X < 0), for  $X \in \mathbb{R}^{n \times n}$  means that the matrix *X* is a real symmetric positive definite (respectively, negative definite). When *x* is a stochastic variable ,  $\mathscr{E}{x}$  stands for the expectation of *x*. The asterisk \* in a matrix is used to denote the term that is induced by symmetry. Matrices, if they are not explicitly stated, are assumed to have compatible dimensions.

#### 2. SYSTEMS DESCRIPTION AND PRELIMINARIES

Fix a probability space  $(\Omega, \mathcal{F}, \mathcal{P})$  and consider the following linear stochastic systems with markovian jump parameters and time-varying delay

$$\Sigma:\begin{cases} \dot{x}(t) = A(r_t)x(t) + A_d(r_t)x(t - \tau(t)) + B(r_t)u(t) \\ x(t) = \phi(t) \quad \forall t \in [-\tau_M, -\tau_m] \end{cases}$$
(1)

where  $x(t) \in \mathbb{R}^n$  is the state vector;  $u(t) \in \mathbb{R}^m$  is the control input;  $\phi(t)$  is the initial condition of x(t);  $\{r_t\}$  is a continuous-time Markovian process with right continuous trajectories and taking values in a finite set  $\mathscr{G} = \{1, 2, ..., \mathcal{N}\}$  with stationary transition probabilities  $\Pi \triangleq \{\pi_{ij}\}$  given by:

$$\operatorname{Prob}\{\theta_{t+h} = j | r_t = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j \\ 1 + \pi_{ii}h + o(h), & i = j \end{cases}$$
(2)

where h>0,  $\lim_{h\to 0} o(h)/h=0$ , and  $\pi_{ij} \ge 0$ , for  $j \ne i$  is the transition rate from mode *i* at time *t* to the mode *j* at time t+h and

$$\pi_{ii} = -\sum_{j=1, j \neq i}^{N} \pi_{ij} \tag{3}$$

The set  $\mathscr{S}$  comprises the various operation modes of system (1) and for each possible value of  $r_t = i \in \mathscr{S}$ , the system matrices of the *i*th mode are denoted by  $A_i$ ,  $B_i$ , and  $A_{di}$ , which are considered here to be real known with appropriate dimensions. It is assumed that the jumping process,  $\{r_t\}$  is accessible, i.e. the operation mode of system ( $\Sigma$ ) is known for every  $t \ge 0$ .

In the system (1), the time delay  $\tau(t)$  is an interval time-varying continuous function satisfying the following assumption:

$$0 \leqslant \tau_m \leqslant \tau(t) \leqslant \tau_M < \infty, \quad \dot{\tau}(t) \leqslant \mu \quad \forall t > 0 \tag{4}$$

where  $\tau_m$  is the lower bound and  $\tau_M$  is the upper bound of the delay  $\tau(t)$ .

#### Remark 1

In practice, the time-varying delay often lies in an interval, in which the lower bound is not 0. Therefore, the introduction of the lower bound  $\tau_m$  will naturally reduce the conservatism. Even for  $\tau_m = 0$ , the criterion may be less conservative than the existing references. This will be demonstrated through numerical examples in the next section.

We consider the following static state feedback controller for the system (1)

$$u_i(t) = K_i x(t) \tag{5}$$

where K is a feedback matrix to be determined.

Let  $u_i^F(t)$  represent the control input after faults have occurred. Then the following fault model is adopted for this study:

$$u_i^F(t) = \Xi u_i(t)$$
  
=  $\sum_{j=1}^m \xi_j H_j K_i x(t)$  (6)

where,  $\Xi = \text{diag}\{\xi_1 \dots \xi_m\}$  with  $\xi_j (j = 1, \dots, m)$  are *m* unrelated random variables. It is assumed that  $\xi_j$  is with mathematical expectation  $\mu_j$  and variance  $\sigma_j^2$ , respectively, and  $H_j = \text{diag}\{\underbrace{0, \dots, 0}_{j-1}, 1, \underbrace{0, \dots, 0}_{m-j}\}$ . For convenience, we also define  $\overline{\Xi} = \text{diag}\{\mu_1, \dots, \mu_m\}$  and  $\Delta = \text{diag}\{\sigma_1, \dots, \sigma_m\}$ .

## Remark 2

Equation (6) describes actuator fault by a random matrix  $\Xi$  that satisfies a certain probabilistic distribution in an interval. In particular, if the case  $\xi_i = 0$ , it stands for an entire missing of signals, and if  $\xi_i = 1$ , it indicates intactness. In fact, actuator signal drift usually occurs in practice situations, while completely failure and intactness are only two special cases.

Combining (1) and (6), we obtain the following close-loop system as follows:

$$\dot{x}(t) = (A_i + B_i \Xi K_i) x(t) + B_i (\Xi - \Xi) K_i x(t) + A_{di} x(t - \tau(t))$$
(7)

For convenience, we define  $A_{1i} = A + B\bar{\Xi}K$ ,  $A_{2i} = B_i(\Xi - \bar{\Xi})K_i$  then (7) can be rewritten as

$$\dot{x}(t) = A_{1i}x(t) + A_{2i}x(t) + A_{di}x(t - \tau(t))$$
(8)

The objective of this study is to develop a reliable controller for the closed-loop system with stochastic fault model described by (6). For this purpose, the following lemmas and definitions are introduced.

#### Lemma 1 ( $Gu \ et \ al. \ [5]$ )

For any constant matrix  $R \in \mathbb{R}^{n \times n}$ , R > 0, scalars  $\tau_m \leq \tau(t) \leq \tau_M$ , and vector function  $\dot{x} : [-\tau_m, 0] \rightarrow \mathbb{R}^n$  such that the following integration is well defined, it holds that

$$-\tau_m \int_{t-\tau_m}^t \dot{x}(t) R \dot{x}(t) \leqslant \begin{bmatrix} x(t-\tau(t)) \\ x(t-\tau_m) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ x(t-\tau_m) \end{bmatrix}$$
(9)

Lemma 2 (Tian and Peng [21])

Suppose M, N, and  $\Omega$  are constant matrices of appropriate dimensions. Then

$$(\tau(t) - \tau_m)M + (\tau_M - \tau(t))N + \Omega < 0 \tag{10}$$

is true for any  $\tau(t) \in [\tau_m \ \tau_M]$  if and only if

$$(\tau_M - \tau_m)M + \Omega < 0 \tag{11}$$

$$(\tau_M - \tau_m)N + \Omega < 0 \tag{12}$$

Remark 3

It will be shown that Lemmas 1 and 2 play key roles in the derivation of a criterion in this paper, which will lead to less conservation.

#### Definition 1

The system (8) is said to be exponentially mean-square stable (EMSS), if there exist constants  $\alpha > 0, \lambda > 0$ , such that t > 0

$$\mathscr{E}\{\|x(t)\|^2\} \leqslant \alpha e^{-\lambda t} \sup_{-\tau_M < s < 0} \{\|\phi(s)\|^2\}$$
(13)

Definition 2

For a given function  $V: C^b_{F_0}([-\tau_M, 0], \mathbb{R}^n) \times S$ , its infinitesimal operator  $\mathscr{L}$  [22] is defined as

$$\mathscr{L}V(x_t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} [\mathscr{E}(V(x_{t+\Delta}|x_t) - V(x_t))]$$
(14)

#### 3. MAIN RESULT

In this section, we aim to develop an innovative approach to guarantee that the system (8) is exponentially mean-square stable. The controller  $K_i$  could be solved from the following results.

### Theorem 1

For given scalars  $\tau_m$ ,  $\tau_M$ ,  $\mu_l$ ,  $\sigma_l$  (l=1,...,m), and matrix  $K_i$ , the system (1) with the actuator fault model (6) is EMSS, if there exist positive-definite matrices  $P_i > 0$ ,  $T_i > 0$ ,  $R_i > 0$ ,  $Q_1 > 0$ ,  $Q_2 > 0$ ,  $S_1 > 0$ ,  $S_2 > 0$ ,  $M_{ik}$ , and  $N_{ik}$ ,  $i \in \mathcal{S}$ ,  $k \in \{1, 2, 3, 4\}$  of appropriate dimensions, such that matrices inequalities (15) and (16) hold.

$$\Pi_{ijk}^{l} = \begin{bmatrix} \Omega_{ij} & \mathscr{A}_{i}^{\mathrm{T}} & \mathscr{H}_{i}^{\mathrm{T}} & \Upsilon_{ik}^{l} \\ * & -\mathscr{R}^{-1} & 0 & 0 \\ * & * & -\widetilde{\mathscr{R}}^{-1} & 0 \\ * & * & * & -R_{2} \end{bmatrix} < 0 \quad (i, j \in \mathscr{S}; k = 1, 2, 3, 4; l = 1, 2)$$
(15)

Copyright © 2010 John Wiley & Sons, Ltd.

$$\sum_{j=1}^{N} \pi_{ij} T_j < S_r \quad (r = 1, 2; i, j \in \mathscr{S})$$
(16)

where

$$\Omega_{ij} = \begin{bmatrix} \Gamma_{1ij} & R_1 + M_{i1} & P_i A_{di} - M_{i1} + N_{i1} & -N_{i1} \\ * & Q_2 - Q_1 - R_1 + M_{i2} + M_{i2}^{\mathrm{T}} & M_{i3}^{\mathrm{T}} - M_{i2} + N_{i2} & M_{i4}^{\mathrm{T}} - N_{i2} \\ * & * & -(1 - \mu)T_i - M_{i3} - M_{i3}^{\mathrm{T}} + N_{i3} + N_{i3}^{\mathrm{T}} & -M_{i4}^{\mathrm{T}} + N_{i4}^{\mathrm{T}} - N_{i3} \\ * & * & * & -Q_2 - N_{i4} - N_{i4}^{\mathrm{T}} \end{bmatrix}$$

$$\begin{split} \Gamma_{1ij} &= P_i A_{1i} + A_{1i}^T P_i + \sum_{j=1}^N \pi_{ij} P_j + T_i + Q_1 - R_1 + \tau_m S_1 + (\tau_M - \tau_m) S_2 \\ \\ & \mathcal{H}_i = \begin{bmatrix} \sigma_1 B_i H_1 K_i \\ \vdots \\ \sigma_k B_i H_k K_i \\ \vdots \\ \sigma_m B_i H_m K_i \end{bmatrix}_{mn \times n} 0_{mn \times n} 0_{mn \times n} \\ \\ & \Upsilon_{ik}^1 = \sqrt{\tau_M - \tau_m} M_{ik} \quad (i \in \mathcal{S}; k = 1, 2, 3, 4) \\ & \Upsilon_{ik}^2 = \sqrt{\tau_M - \tau_m} N_{ik} \quad (i \in \mathcal{S}; k = 1, 2, 3, 4) \\ & \mathcal{H} = \tau_m^2 R_1 + (\tau_M - \tau_m) R_2 \\ & \widetilde{\mathcal{H}} = \text{diag}\{\underbrace{\mathcal{H}_{\dots,\mathcal{H}}}_m\} \\ & \mathcal{A}_i = [A_i + B_i \overline{\Xi} K_i \ 0 \ A_{di} \ 0] \end{split}$$

Proof

Construct a Lyapunov-Krasovskii functional candidate as

$$V(x_{t}, r_{t}) = \sum_{i=1}^{4} V_{i}(x_{t}, r_{t})$$

$$V_{1}(x_{t}, r_{t}) = x^{T}(t)P(r_{t})x(t)$$

$$V_{2}(x_{t}, r_{t}) = \int_{t-\tau(t)}^{t} x^{T}(s)T(r_{t})x(s) ds + \int_{t-\tau_{m}}^{t} x^{T}(s)Q_{1}x(s) ds + \int_{t-\tau_{M}}^{t-\tau_{m}} x^{T}(s)Q_{2}x(s) ds$$

$$V_{3}(x_{t}, r_{t}) = \tau_{m} \int_{-\tau_{m}}^{0} \int_{t+s}^{t} \dot{x}^{T}(v)R_{1}\dot{x}(v) dv ds + \int_{-\tau_{M}}^{-\tau_{m}} \int_{t+s}^{t} \dot{x}^{T}(v)R_{2}\dot{x}(v) dv ds$$

$$V_{4}(x_{t}, r_{t}) = \int_{-\tau_{m}}^{0} \int_{t+s}^{t} x^{T}(v)S_{1}x(v) dv ds + \int_{-\tau_{M}}^{-\tau_{m}} \int_{t+s}^{t} x^{T}(v)S_{2}x(v) dv ds$$

From the definition of  $\Xi$  and  $\overline{\Xi}$ , we can easily know

$$\mathscr{E}[B_i(\Xi - \bar{\Xi})K_i] = 0 \tag{17}$$

Copyright © 2010 John Wiley & Sons, Ltd.

Also, we can have (18) from the definition  $\Xi$  in (6)

$$[B_i(\Xi - \bar{\Xi})K_i]^{\mathrm{T}}\mathscr{R}[B_i(\Xi - \bar{\Xi})K_i] = \sum_{j=1}^m \sigma_j^2 K^{\mathrm{T}} C_i^{\mathrm{T}} B^{\mathrm{T}} \mathscr{R} B_i H_j K_i$$
(18)

Using Lemma 1 and the infinitesimal operator (14) for system (8), we have

$$\begin{aligned} \mathscr{L}V_{1}(x_{t}, i, t) &= 2x^{\mathrm{T}}(t)P_{i}\left[\left(A_{1i} + \frac{1}{2}\sum_{j=1}^{N}\pi_{ij}P_{j}\right)x(t) + A_{di}x(t-\tau(t))\right] \tag{19} \\ &= \mathcal{L}V_{2}(x_{t}, i, t) \leqslant x^{\mathrm{T}}(t)(T_{i}+Q_{1})x(t) - (1-\mu)x^{\mathrm{T}}(t-\tau(t))T_{i}x(t-\tau(t))) \\ &+ \int_{t-\tau(t)}^{t}x^{\mathrm{T}}(s)\sum_{j=1}^{N}\pi_{ij}T_{j}x(s)\,\mathrm{d}s \\ &+ x^{\mathrm{T}}(t-\tau_{m})(Q_{2}-Q_{1})x(t-\tau_{m}) - x^{\mathrm{T}}(t-\tau_{M})Q_{2}x(t-\tau_{M}) \tag{20} \end{aligned} \\ \\ &= \mathcal{L}V_{3}(x_{t}, i, t) &= \dot{x}^{\mathrm{T}}(t)\mathscr{R}\dot{x}(t) - \tau_{m}\int_{t-\tau_{m}}^{t}\dot{x}^{\mathrm{T}}(s)R_{1}\dot{x}(s)\,\mathrm{d}s - \int_{t-\tau_{M}}^{t-\tau_{m}}\dot{x}^{\mathrm{T}}(s)R_{2}\dot{x}(s)\,\mathrm{d}s \\ &\leqslant \xi^{\mathrm{T}}(t)\mathscr{A}_{i}^{\mathrm{T}}\mathscr{R}\mathscr{A}_{i}\xi(t) + x^{\mathrm{T}}(t)\sum_{j=1}^{m}[\sigma_{j}B_{i}H_{j}K_{i}]^{\mathrm{T}}\mathscr{R}[\sigma_{j}B_{i}H_{j}K_{i}]x(t) \\ &+ \left[x(t)\\x(t-\tau_{m})\right]^{\mathrm{T}}\left[-R_{1} \quad R_{1}\\R_{1} \quad -R_{1}\right]\left[x(t)\\x(t-\tau_{m})\right] \tag{21} \end{aligned}$$

and employing the free-weighing matrix method [23, 24]

$$2\xi^{\mathrm{T}}(t)M_{ik}\left[x(t-\tau_m) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_m} \dot{x}(s)\,\mathrm{d}s\right] = 0$$
(23)

$$2\xi^{\mathrm{T}}(t)N_{ik}\left[x(t-\tau(t)) - x(t-\tau_{M}) - \int_{t-\tau_{M}}^{t-\tau(t)} \dot{x}(s)\,\mathrm{d}s\right] = 0$$
(24)

$$-2\xi^{\mathrm{T}}(t)M_{ik}\int_{t-\tau(t)}^{t-\tau_{m}}\dot{x}(s)\mathrm{d}s \leqslant (\tau(t)-\tau_{m})\xi^{\mathrm{T}}(t)M_{ik}R_{2}^{-1}M_{ik}^{\mathrm{T}}\xi(t) + \int_{t-\tau(t)}^{t-\tau_{m}}\dot{x}^{\mathrm{T}}(s)R_{2}\dot{x}(s)\mathrm{d}s$$
(25)

$$-2\xi^{\mathrm{T}}(t)N_{ik}\int_{t-\tau_{M}}^{t-\tau(t)}\dot{x}(s)\,\mathrm{d}s \leqslant (\tau_{M}-\tau(t))\xi^{\mathrm{T}}(t)N_{ik}R_{2}^{-1}N_{ik}^{\mathrm{T}}\xi(t) + \int_{t-\tau_{M}}^{t-\tau(t)}\dot{x}^{\mathrm{T}}(s)R_{2}\dot{x}(s)\,\mathrm{d}s$$
(26)

Copyright © 2010 John Wiley & Sons, Ltd.

Define  $\sum_{j=1}^{N} \lambda_{ij} T_j = \mathbb{T}$ , from (16), it has

$$\int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s) \mathbb{T}x(s) \,\mathrm{d}s - \int_{t-\tau_{m}}^{t} x^{\mathrm{T}}(s) S_{1}x(s) \,\mathrm{d}s - \int_{t-\tau_{M}}^{t-\tau_{m}} x^{\mathrm{T}}(s) S_{2}x(s) \,\mathrm{d}s$$

$$= \int_{t-\tau_{m}}^{t} x^{\mathrm{T}}(s) (\mathbb{T} - S_{1})x(s) \,\mathrm{d}s + \int_{t-\tau(t)}^{t-\tau_{m}} x^{\mathrm{T}}(s) \mathbb{T}x(s) \,\mathrm{d}s - \int_{t-\tau_{M}}^{t-\tau_{m}} x^{\mathrm{T}}(s) S_{2}x(s) \,\mathrm{d}s$$

$$\leqslant \int_{t-\tau_{m}}^{t} x^{\mathrm{T}}(s) (\mathbb{T} - S_{1})x(s) \,\mathrm{d}s + \int_{t-\tau(t)}^{t-\tau_{m}} x^{\mathrm{T}}(s) (\mathbb{T} - S_{2})x(s) \,\mathrm{d}s \leqslant 0$$
(27)

Combining (19)–(27), we have

$$\mathscr{L}V(x_t, i, t) \leq \zeta^{\mathrm{T}}(t) [\Omega_{ij} + \mathscr{A}_i^{\mathrm{T}} \mathscr{R} \mathscr{A}_i + \mathscr{H}^{\mathrm{T}} \mathscr{R} \mathscr{H} + (\tau(t) - \tau_m)(t) M_{ik} R_2^{-1} M_{ik}^{\mathrm{T}} + (\tau_M - \tau(t)) N_{ik} R_2^{-1} N_{ik}^{\mathrm{T}}] \zeta(t)$$
(28)

where  $\zeta(t) = [x^{\mathrm{T}}(t) \ x^{\mathrm{T}}(t-\tau_m) \ x^{\mathrm{T}}(t-\tau(t)) \ x^{\mathrm{T}}(t-\tau_M)]^{\mathrm{T}}$ ,  $\mathcal{R}, \mathcal{H}, \mathcal{A}_i$ , and  $\Omega_{ij}$  are defined in Theorem 1.

Using Schur complements and Lemma 2, it can be shown that (15) is the sufficient condition for guaranteeing

$$\Omega_{ij} + \mathscr{A}_i^{\mathrm{T}} \mathscr{R} \mathscr{A}_i + \mathscr{H}^{\mathrm{T}} \mathscr{R} \mathscr{H} + (\tau(t) - \tau_m)(t) M_{ik} R_2^{-1} M_{ik}^{\mathrm{T}} + (\tau_M - \tau(t)) N_{ik} R_2^{-1} N_{ik}^{\mathrm{T}} < 0$$
<sup>(29)</sup>

Then, the following inequality can be concluded

$$\mathscr{E}\{\mathscr{L}V(x_t, i, t)\} < -\lambda_{\min}(\Pi^l_{ijk}) \mathscr{E}\{\zeta^{\mathrm{T}}(t)\zeta(t)\}$$
(30)

Define a new function as

$$W(x_t, i, t) = e^{\varepsilon t} V(x_t, i, t)$$
(31)

Its infinitesimal operator  $\mathscr{L}$  is given by

$$\mathscr{W}(x_t, i, t) = \varepsilon e^{\varepsilon t} V(x_t, i, t) + e^{\varepsilon t} \mathscr{L} V(x_t, i, t)$$
(32)

By the generalized Itô formula [22], we can obtain from (32) that

$$\mathscr{E}\{W(x_t, i, t)\} - \mathscr{E}\{W(x_0, i)\} = \int_0^t \varepsilon e^{\varepsilon s} \mathscr{E}\{V(x_s, i)\} ds + \int_0^t e^{\varepsilon s} \mathscr{E}\{\mathscr{L}V(x_s, i)\} ds$$
(33)

Then, using the similar method of [25], we can see that there exists a positive number  $\alpha$  such that for t>0

$$\mathscr{E}\{V(x_t, i, t)\} \leqslant \alpha \sup_{-\tau_M \leqslant s \leqslant 0} \{\|\phi(s)\|^2\} e^{-\varepsilon t}$$
(34)

since  $V(x_t, i, t) \ge \{\lambda_{\min}(P_i)\} x^{\mathrm{T}}(t) x(t)$ , it can be shown from (34) that for  $t \ge 0$ 

$$\mathscr{E}\{x^{\mathrm{T}}(t)x(t)\} \leqslant \bar{\alpha}^{-\varepsilon t} \sup_{-\tau_M \leqslant s \leqslant 0} \{\|\phi(s)\|^2\}$$
(35)

where  $\bar{\alpha} = \alpha / (\lambda_{\min} P_i)$ . Recalling Definition 1, the proof can be completed.

#### Remark 4

Theorem 1 provides a delay-dependent stochastic stability condition for MJS with interval timevarying delay. The convexity of the matrix functions is used to avoid the conservative caused by enlarging  $\tau(t)$  to  $\tau_M$  in the deriving results.

Copyright © 2010 John Wiley & Sons, Ltd.

Remark 5

By introducing the item  $\int_{t-\tau(t)}^{t} x^{T}(s)T(r_{t})x(s) ds$  in  $V_{2}(x_{t}, r_{t})$ , and splitting its weak infinitesimal operator into two parts, it will lead to less conservative of Theorem 1, which will be demonstrated through numerical examples in the next section.

If  $B_i = 0, (i \in \mathcal{S})$ , then the system (1) can be converted as an unforce system

$$\dot{x}(t) = A(r_t)x(t) + A_d(r_t)x(t - \tau(t))$$

$$x(t) = \phi(t) \quad \forall t \in [-\tau_M, -\tau_m]$$
(36)

The following result can be concluded directly from Theorem 1.

#### Corollary 1

For given scalars  $\tau_m$ ,  $\tau_M$ ,  $\mu_l$ , and  $\sigma_l(l=1,...,m)$ , the system (36) is EMSS, if there exist positivedefinite matrices  $P_i > 0$ ,  $T_i > 0$ ,  $R_i > 0$ ,  $Q_1 > 0$ ,  $Q_2 > 0$ ,  $S_1 > 0$ ,  $S_2 > 0$ ,  $M_{ik}$ , and  $N_{ik}$ ,  $i \in \mathcal{S}$ ,  $k \in \{1, 2, 3, 4\}$ of appropriate dimensions, such that LMIs (37) and (38) hold.

$$\begin{bmatrix} \bar{\Omega}_{ij} & \bar{\mathscr{A}}_i^{\mathrm{T}} & \Upsilon_{ik}^l \\ * & -\mathscr{R} & 0 \\ * & * & 0 - R_2 \end{bmatrix} < 0 \quad (i, j \in \mathscr{S}; k = 1, 2, 3, 4; l = 1, 2)$$
(37)

$$\sum_{j=1}^{N} \pi_{ij} T_j < S_r \quad (r=1,2; i, j \in \mathscr{S})$$

$$(38)$$

where

$$\bar{\Omega}_{ij} = \begin{bmatrix} \bar{\Gamma}_{1ij} & R_1 + M_{i1} & P_i A_{di} - M_{i1} + N_{i1} & -N_{i1} \\ * & Q_2 - Q_1 - R_1 + M_{i2} + M_{i2}^T & M_{i3}^T - M_{i2} + N_{i2} & M_{i4}^T - N_{i2} \\ * & * & -(1 - \mu)T_i - M_{i3} - M_{i3}^T + N_{i3} + N_{i3}^T & -M_{i4}^T + N_{i4}^T - N_{i3} \\ * & * & * & -Q_2 - N_{i4} - N_{i4}^T \end{bmatrix}$$
  
$$\bar{\Gamma}_{1ij} = P_i A_i + A_i^T P_i + \sum_{j=1}^N \pi_{ij} P_j + T_i + Q_1 - R_1 + \tau_m S_1 + (\tau_M - \tau_m) S_2$$
  
$$\bar{\mathcal{A}}_i = [A_i \mathcal{R} \ 0 \ A_{di} \mathcal{R} \ 0]$$

Now, we are in a position to state a delay-dependent reliable control for the system (8) based on Theorem 1.

#### Theorem 2

For prescribed  $\mu_l$ ,  $\sigma_l$  (l=1,...,m) and given scalars  $\varepsilon$ ,  $\tau_m$ ,  $\tau_M$ , the system (1) with the faulty actuator (6) is EMSS if there exist positive-definite matrices  $X_i > 0$ ,  $\hat{T}_i > 0$ ,  $\hat{R}_i > 0$ ,  $\hat{Q}_1 > 0$ ,  $\hat{Q}_2 > 0$ ,  $\hat{S}_1 > 0$ ,  $\hat{S}_2 > 0$ ,  $\hat{M}_{ik}$ ,  $\hat{N}_{ik}$ ,  $i \in \mathcal{S}$ ,  $k \in \{1, 2, 3, 4\}$  and  $Y_i$  of appropriate dimensions, such that LMIs (39) and (40) hold. Furthermore, the reliable controller gain  $K_i = Y_i X_i^{-1}$ .

$$\begin{bmatrix} \hat{\Omega}_{i} & \hat{\mathscr{A}}_{i}^{\mathrm{T}} & \hat{\mathscr{H}}_{i}^{\mathrm{T}} & \hat{\Upsilon}_{ik}^{l} \\ * & -2\varepsilon X_{i} + \varepsilon^{2} \hat{\mathscr{R}} & 0 & 0 \\ * & * & -2\varepsilon \tilde{X}_{i} + \varepsilon^{2} \hat{\tilde{\mathscr{R}}} & 0 \\ * & * & -2\varepsilon \tilde{X}_{i} + \varepsilon^{2} \hat{\tilde{\mathscr{R}}} & 0 \\ * & * & & -\hat{R}_{2} \end{bmatrix} < 0 \quad (i, j \in \mathscr{G}, i \neq j; ; k = 1, 2, 3, 4, l = 1, 2) \quad (39)$$

$$\sum_{j=1}^{N} \pi_{ij} \hat{T}_{j} < \hat{S}_{r} \quad (r = 1, 2; i, j \in \mathscr{G}) \quad (40)$$

Copyright © 2010 John Wiley & Sons, Ltd.

where

$$\hat{\Omega}_{i} = \begin{bmatrix} \tilde{\Gamma}_{1i} & \hat{R}_{1} + \hat{M}_{i1} & A_{di}X_{i} - \hat{M}_{i1} + \hat{N}_{i1} & -\hat{N}_{i1} & \pi_{ij}\mathscr{X}_{i} \\ * & \hat{Q}_{2} - \hat{Q}_{1} - \hat{R}_{1} + \hat{M}_{i2} + \hat{M}_{i2}^{\mathrm{T}} & \hat{M}_{i3}^{\mathrm{T}} - \hat{M}_{i2} + \hat{N}_{i2} & \hat{M}_{i4}^{\mathrm{T}} - \hat{N}_{i2} & 0 \\ * & * & -(1-\mu)\hat{T}_{i} - \hat{M}_{i3} - \hat{M}_{i3}^{\mathrm{T}} + \hat{N}_{i3} + \hat{N}_{i3}^{\mathrm{T}} & -\hat{M}_{i4}^{\mathrm{T}} + \hat{N}_{i4}^{\mathrm{T}} - \hat{N}_{i3} & 0 \\ * & * & & & -\hat{Q}_{2} - \hat{N}_{i4} - \hat{N}_{i4}^{\mathrm{T}} & 0 \\ * & * & & & & & & -\pi_{ii}\tilde{\mathscr{X}} \end{bmatrix}$$

$$\hat{\Gamma}_{1i} = A_i X_i + X_i A_i^{\mathrm{T}} + B_i \Xi Y_i + Y_i^{\mathrm{T}} \Xi^{\mathrm{T}} B_i^{\mathrm{T}} + \hat{T}_i + \hat{Q}_1 - \hat{R}_1 + \tau_m \hat{S}_1 + (\tau_M - \tau_m) \hat{S}_2 + \pi_{ii} X_i$$

$$\hat{\mathscr{H}}_{i} = \begin{bmatrix} \varepsilon_{1}B_{i}H_{1}Y_{i} \\ \vdots \\ \varepsilon_{k}B_{i}H_{k}Y_{i} \\ \vdots \\ \varepsilon_{m}B_{i}H_{m}Y_{i} \end{bmatrix}_{mn \times n} 0_{mn \times n} \quad 0_{mn \times n} \\ \hat{\Upsilon}_{ik}^{1} = \sqrt{\tau_{M} - \tau_{m}}\hat{M}_{ik} \quad (i \in \mathscr{S}; k = 1, 2, 3, 4) \\ \hat{\Upsilon}_{ik}^{2} = \sqrt{\tau_{M} - \tau_{m}}\hat{N}_{ik} \quad (i \in \mathscr{S}; k = 1, 2, 3, 4) \\ \hat{\mathscr{R}} = \tau_{m}^{2}\hat{R}_{1} + (\tau_{M} - \tau_{m})\hat{R}_{2} \\ \hat{\mathscr{A}}_{i} = [A_{i}X + B_{i}\bar{\Xi}Y_{i} \quad 0 \quad A_{di}X \quad 0] \\ \hat{\mathscr{X}}_{i} = [X_{i}, \dots, X_{i}] \\ \tilde{\mathscr{X}} = \operatorname{diag}\{X_{1}, \dots, X_{i-1}, X_{i+1}, \dots, X_{N}\} \\ \tilde{X}_{i} = \operatorname{diag}\{X_{i}, \dots, X_{i}\} \\ \tilde{\mathscr{M}} = \operatorname{diag}\{\hat{\mathscr{M}}, \dots, \hat{\mathscr{M}}\} \\ \end{bmatrix}$$

Proof

By Schur complement, the matrix inequality (15) holds if and only if

$$\begin{bmatrix} \check{\boldsymbol{\Omega}}_{ij} & \mathscr{A}_i^{\mathrm{T}} P_i & \mathscr{H}_{ik}^{\mathrm{T}} P_i & \sqrt{\tau_M - \tau_m} M \\ * & -P_i \mathscr{R}^{-1} P_i & 0 & 0 \\ * & * & -P_i \mathscr{R}^{-1} P_i & 0 \\ * & * & * & -R_2 \end{bmatrix} < 0$$
(41)

where

$$\check{\Omega}_{ij} = \begin{bmatrix} \check{\Gamma}_{1ij} & R_1 + M_{i1} & P_i A_{di} - M_{i1} + N_{i1} & -N_{i1} & \pi_{ij} \mathscr{P}_i \end{bmatrix}$$

$$\check{\Omega}_{ij} = \begin{bmatrix} \check{\Gamma}_{1ij} & R_1 + M_{i1} & P_i A_{di} - M_{i1} + N_{i1} & -N_{i1} & \pi_{ij} \mathscr{P}_i \end{bmatrix}$$

$$* \quad Q_2 - Q_1 - R_1 + M_{i2} + M_{i2}^T & M_{i3}^T - M_{i2} + N_{i2} & M_{i4}^T - N_{i2} & 0 \\ * \quad * & -(1 - \mu)T_i - M_{i3} - M_{i3}^T + N_{i3} + N_{i3}^T & -M_{i4}^T + N_{i4}^T - N_{i3} & 0 \\ * \quad * & * & -Q_2 - N_{i4} - N_{i4}^T & 0 \\ * & * & * & * & -\pi_{ij} \widetilde{\mathscr{P}} \end{bmatrix}$$

$$\check{\Gamma}_{1ij} = P_i A_i + A_i^{\rm T} P_i + \pi_{ii} P_i + T_i + Q_1 - R_1 + \tau_m S_1 + (\tau_M - \tau_m) S_2$$

Copyright © 2010 John Wiley & Sons, Ltd.

$$\mathcal{P}_i = [\underbrace{P_i, \dots, P_i}_{N-1}]$$
$$\tilde{\mathcal{P}} = \text{diag}\{P_1, \dots, P_{i-1}, P_{i+1}, \dots, P_N\}$$

Owing to

$$(\mathscr{R} - \varepsilon^{-1} P_i)^{-1} (\mathscr{R} - \varepsilon^{-1} P_i) \ge 0 \tag{42}$$

which gives

$$-P_i \mathscr{R}^{-1} P_i \leqslant -2\varepsilon P_i + \varepsilon^2 \mathscr{R} \tag{43}$$

we have that (41) holds if

$$\begin{bmatrix} \breve{\Omega}_{ij} & \mathscr{A}_i^{\mathrm{T}} P_i & \mathscr{H}_{ik}^{\mathrm{T}} P_i & \sqrt{\tau_M - \tau_m} M \\ * & -2\varepsilon P_i + \varepsilon^2 \mathscr{R} & 0 & 0 \\ * & * & -2\varepsilon \tilde{P}_i + \varepsilon^2 \widetilde{\mathscr{R}} & 0 \\ * & * & * & -R_2 \end{bmatrix} < 0$$
(44)

Defining  $X_i = P_i^{-1}$ , and applying the congruence transformation diag $\{X_i, X_i, X_i, X_i, \tilde{X}_i, X_i, \tilde{X}_i, \tilde{X}_i,$ 

## Remark 6

The inequality (43) is used to bound the term  $-P_i \mathscr{R}^{-1} P_i$  in (41). This step can be improved by adopting the cone complementary algorithm [26], which is popular in recent control designs. The scaling parameter  $\varepsilon > 0$  can be used to improve conservatism in Theorem 2.

#### Remark 7

From Theorem 2, it can be seen that the solvability of LMIs (39) and (40) depends on the distribution of the actuator fault taking value. More information are taken into account in our results comparing with the usual fault modeling method in existing results.

## 4. ILLUSTRATIVE EXAMPLES

In this section, well-studied examples are used to illustrate the effectiveness of the approaches proposed in this paper.

Example 1

Consider a Markovian jump system in (36) with two modes and the following parameters [27]:

$$A_{1} = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & -3.2684 \end{bmatrix}, \quad A_{2} = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix}$$
$$A_{d1} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -2.8306 & -0.4978 \\ -0.8436 & -1.0115 \end{bmatrix}$$

Table I shows the comparative results with assumption that  $\pi_{22} = -0.8$ . Obviously, for the same conditions for the time delay, our method can lead to less conservative results, and the criterion can be applied to interval time-varying delays (Figures 1 and 2), that is, it is not needed that  $\tau_m = 0$  and  $\mu < 1$ .

To illustrate the proposed method on reliable control, another example is considered as follows.

# Example 2

Consider a MJS in (1) with two modes and following parameters:

$$A_{1} = \begin{bmatrix} -1 & 0.5 \\ -0.2 & -1 \end{bmatrix}, \quad A_{2} = \begin{bmatrix} 1.8 & 0.8 \\ 0.15 & -2 \end{bmatrix}$$
$$A_{d1} = \begin{bmatrix} -0.25 & 0 \\ 0 & -0.2 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.12 & -0.1 \\ 0.1 & -0.11 \end{bmatrix}$$
$$B_{1} = \begin{bmatrix} 0.5 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad B_{2} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad \pi_{11} = -3, \pi_{22} = -1$$

 $0 < \tau(t) < 0.7.$ 

Two cases are considered as follows:

*Case 1*: We assume the actuators are normal, that is, The parameter  $\Xi$  of fault model (6) has expectation and variance

$$\bar{\Xi} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Table I. Maximum allowable values of  $\tau_M$  with given  $\pi_{22} = -0.8$ .

π <sub>11</sub>	-0.2	-1.5
$[27] (\mu = 0)$	0.635	0.534
Corollary 1 ( $\mu = 0$ )	0.635	0.534
$[27]$ ( $\mu = 0.8$ )	0.424	0.414
Corollary 1 ( $\mu = 0.8, \tau_m = 0$ )	0.496	0.483
$[27]$ ( $\mu = 1.3$ )	0.386	0.385
Corollary 1 ( $\mu = 1.3, \tau_m = 0$ )	0.492	0.492
Corollary 1 ( $\mu = 0.8, \tau_m = 0.1$ )	0.496	0.484
Corollary 1 ( $\mu = 1.3, \tau_m = 0.1$ )	0.493	0.484



Figure 1. Operation modes.



Figure 2. Interval time-varying delay.



Figure 3. Standard controller without failure.

and

$$\Delta = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

respectively.

Selecting  $\varepsilon = 1$ , according to Theorem 2, we obtain the controllers called standard controller as followers:

$$K_1 = \begin{bmatrix} 0.7681 & -0.9934 \\ -2.0327 & -3.8874 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -9.1930 & -2.5499 \\ -1.9576 & 11.4326 \end{bmatrix}$$
(45)

Case 2: Assuming the admissible set of actuator faults are given by

$$\bar{\Xi} = \begin{bmatrix} 0.2 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\Delta = \begin{bmatrix} 0 & 0 \\ 0 & 0.2 \end{bmatrix}$$



Figure 4. Reliable controller without failure.



Figure 5. Standard controller with failure.

that is, there exist some actuator faults, such that actuator drifts and fluctuations occur. Selecting  $\varepsilon = 1$ , according to Theorem 2, we get the controllers called reliable controllers.

$$K_1 = \begin{bmatrix} 3.8404 & -4.9672 \\ -2.0327 & -3.8874 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -45.9652 & -12.7497 \\ -1.9576 & 11.4326 \end{bmatrix}$$
(46)

Assuming the initial conditions  $\phi(t) = [-1 \ 1]^T$ ,  $t \in [0, 0.7]$ . Figures 3 and 4 show the state response for normal situation using the standard controller and the reliable controller, respectively. It is clear that the two controllers perform very satisfactorily when no failures occur. When actuators are abnormal, the state responses for the standard and the reliable controllers are shown in Figures 5 and 6, respectively. It is observed that when actuator failures occur, the closed-loop system with the standard controller is not even asymptotically stable, while the closed-loop system using the reliable controller still operates well and maintains an acceptable level of performance.



Figure 6. Reliable controller with failure.

### 5. CONCLUSION

In this paper, we concentrate on the reliable control design problem for interval time-varying delay systems with Markovian jumping parameters. Based on a new practical actuator fault model, a reliable control design methodology is presented to achieve a less conservative result, not only when the system is operating properly, but also in the presence of certain actuator failures. Numerical examples are given to illustrate the design procedures.

#### ACKNOWLEDGEMENTS

The authors acknowledge Natural Science Foundation of China (NSFC) for its support under grant numbers 60704024, 60904013 and 60774060.

#### REFERENCES

- 1. Wang Z, Lam J, Liu X. Exponential filtering for uncertain Markovian jump time-delay systems with nonlinear disturbances. *IEEE Transactions on Circuits and Systems II: Express Briefs* 2004; **51**(5):262–268.
- 2. Hu L, Shi P, Huang B.  $H_{\infty}$  control for sampled-data linear systems with two Markov processes. *Optimal Control Applications and Methods* 2005; **26**(6):291–306.
- Feng J, Lam J, Xu S, Shu Z. Optimal stabilizing controllers for linear discrete-time stochastic systems. Optimal Control Applications and Methods 2008; 29(3):243–253.
- 4. De Souza C, Li X. Delay-dependent robust  $H_{\infty}$  control of uncertain linear state-delayed systems. Automatica 1999; **35**(7):1313–1321.
- 5. Gu K, Kharitonov V, Chen J. Stability and Robust Stability of Time-delay Systems, 2003.
- 6. Peng C, Tian Y, Tade M. State feedback controller design of networked control systems with interval time-varying delay and nonlinearity. *International Journal of Robust and Nonlinear Control* 2008; **18**(12):1285–1301.
- 7. Peng C, Tian Y, Tian E. Improved delay-dependent robust stabilization conditions of uncertain T–S fuzzy systems with time-varying delay. *Fuzzy Sets and Systems* 2008; **159**(20):2713–2729.
- Benjelloun K, Boukas E. Mean square stochastic stability of linear time-delay system with Markovian jumping parameters. *IEEE Transactions on Automatic Control* 1998; 43(10):1456–1460.
- 9. Mao X. Exponential stability of stochastic delay interval systems with Markovian switching. *IEEE Transactions* on Automatic Control 2002; **47**(10):1604–1612.
- Xu S, Chen T, Lam J. Robust filtering for uncertain Markovian jump systems with mode-dependent time delays. *IEEE Transactions on Automatic Control* 2003; 48(5):901.
- 11. Boukas E, Liu Z, Shi P. Delay-dependent stability and output feedback stabilisation of Markov jump system with time-delay. *IEE Proceedings—Control Theory and Applications* 2002; **149**:379.
- Shu Z, Lam J, Xu S. Robust stabilization of Markovian delay systems with delay-dependent exponential estimates. *Automatica* 2006; 42(11):2001–2008.

- 13. Chen W, Guan Z, Yu P. Delay-dependent stability and H control of uncertain discrete-time Markovian jump systems with mode-dependent time delays. *Systems and Control Letters* 2004; **52**(5):361–376.
- 14. Vidyasagar M, Viswanadham N. Reliable stabilization using a multi-controller configuration. *Automatica* 1985; **21**(5):599–602.
- 15. Yang Y, Yang G, Soh Y. Reliable control of discrete-time systems with actuator failure. *IEE Proceedings—Control Theory and Applications* 2000; **147**:428–432.
- 16. Shi P, Boukas E, Nguang S, Guo X. Robust disturbance attenuation for discrete-time active fault tolerant control systems with uncertainties. *Optimal Control Applications and Methods* 2003; **24**(2):85–101.
- 17. Wang F, Zhang Q, Yao B. LMI-based reliable  $H_{\infty}$  filtering with sensor failure. *International Journal of Innovative Computing, Information and Control* 2005; **2**(4):737–749.
- 18. Yao B, Wang F. LMI approach to reliable  $H_{\infty}$  control of linear systems. Journal of Systems Engineering and Electronics 2006; 17(2):381–386.
- 19. Yang G, Wang J, Soh Y. Reliable H controller design for linear systems. Automatica 2001; 37(5):717-725.
- 20. Kongdm FHJ. Stable fault-tolerance control for a class of networked control systems. Acta Automatica Sinica 2005; **32**(2):267–273.
- Tian E, Peng C. Delay-dependent stability analysis and synthesis of uncertain T–S fuzzy systems with time-varying delay. *Fuzzy Sets and Systems* 2006; 157(4):544–559.
- 22. Mao X. Exponential stability of stochastic delay interval systems with Markovian switching. *IEEE Transactions* on Automatic Control 2002; **47**(10):1604–1612.
- He Y, Wang Q, Lin C, Wu M. Augmented Lyapunov functional and delay-dependent stability criteria for neutral systems. *International Journal of Robust and Nonlinear Control* 2005; 15(18):923–933.
- 24. Yue D, Han Q, Lam J. Network-based robust H control of systems with uncertainty. *Automatica* 2005; **41**(6):999–1007.
- 25. Yue D, Han Q. Delay-dependent exponential stability of stochastic systems with time-varying delay, nonlinearity, and Markovian switching. *IEEE Transactions on Automatic Control* 2005; **50**(2):217–222.
- 26. El Ghaoui L, Oustry F, AitRami M. A cone complementarity linearization algorithm for staticoutput-feedback and related problems. *IEEE Transactions on Automatic Control* 1997; **42**(8):1171–1176.
- 27. Xu S, Lam J, Mao X. Delay-dependent  $H_{\infty}$  control and filtering for uncertain Markovian jump systems with time-varying delays. *IEEE Transactions on Circuits and Systems I: Regular Papers* 2007; **54**(9):2070–2077.